C207; Project Set #6

due 4/25/2013

The Rise of Stromgren Spheres

Consider an O-star that turns on and begins radiating a surrounding medium of neutral hydrogen. This produces an ionized region, initially small but growing in size until the total number of ionizations balance recombinations – i.e., when the radius of the HII region reaches the Stromgren radius, R_s. Here we derive a simple time dependent solution for the growth of an HII region.[1]

We will imagine the HII region to have a relatively sharp outer boundary at radius $R(t)$, which marks the ionization front. Defining $\dot{N}_{\rm ion}$ ($\dot{N}_{\rm rec}$) as the total number of photoionizations (recombinations) per second that occur in the surrounding gas, we have the expression

$$\frac{dN_{\rm tot}}{dt} = \dot{N}_{\rm ion} - \dot{N}_{\rm rec} \tag{1}$$

where $N_{\rm tot}(t)$ is the total number of hydrogen ions contained within the radius $R(t)$. When the rate of ionizations exceeds the rate of recombinations, the number of ionized atoms in the HII region (and hence its radius) will increase with time[2].

For simplicity, we'll assume that the O-star has luminosity L and emits all photons at the ionization threshold – i.e., at a single wavelength $\lambda_0 = 912$ Å. We'll assume the surrounding gas is pure hydrogen with a number density, n, that is constant with radius.

a) Insert expressions for, $\dot{N}_{\rm ion}, \dot{N}_{\rm rec}$ in equation 1 and derive an analytic expression for $R(t)$. What is the final radius of the HII region?

b) What is the timescale for the HII region to grow to its final radius? If the gas has a density $n = 1$ cm^{-3} and a temperature $T = 10^4$ K, does the HII region have time to reach its final radius before the O-star dies?

c) Write down the expression for the velocity of the outer edge (ionization front) of the HII region. If the O-star has a luminosity $L = 10^{39}$ ergs s^{-1}, how does the typical velocity of the HII expansion compare to the sound speed for the above temperature and density?[3]

So far, our arguments have all concerned the global ionization state. Let's consider in more detail the the radial ionization structure inside the HII region once it has reached its final (equilibrium) radius.

[1] We consider here the growth only due to photoionization. Because the surrounding gas is being radiatively heated at the same time, its pressure increases and the ionized region may expand hydrodynamically as well. We will see, however, that this hydrodynamical expansion should occur mainly after we have reached global photoionization balance.

[2] Setting $dN_{\rm tot}/dt = 0$ simply reproduces the Stromgren argument for the size of an HII region in equilibrium.

[3] Supersonic expansion of the ionization front suggests we do not need to consider the hydrodynamical effects just yet. See Chaper 20 of Shu's hydrodynamics book for a detailed description of the various interesting dynamical effects in HII regions.

Although the gas within an HII region is highly ionized, some small fraction of neutral hydrogen remains.

To make the problem more tractable, we'll assume that the radius of the O-star is very small and that there is no appreciable attenuation of the radiation field. This should be OK at least in the inner parts of the HII region. Ignore any re-emission or scattering of ionizing photons in the HII region.

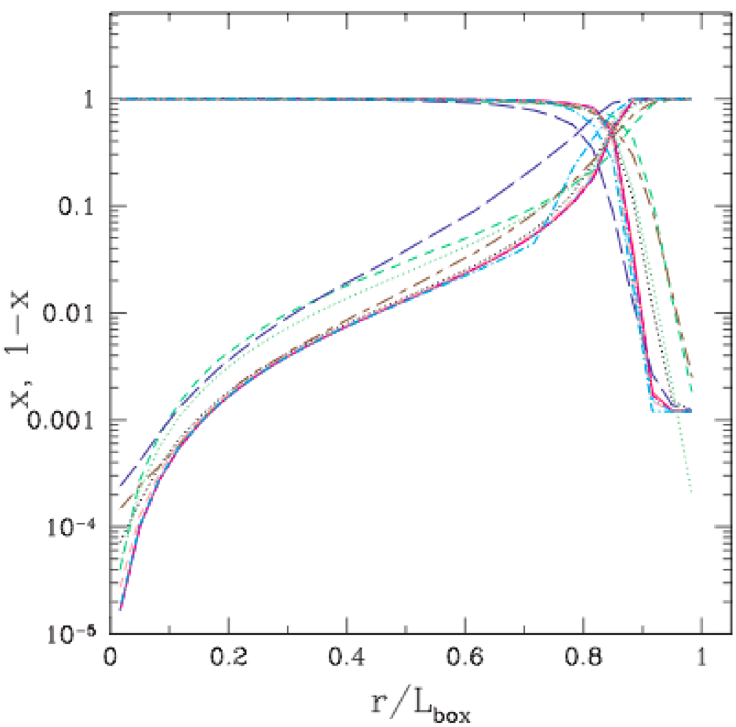

Figure 1: The fraction of neutral hydrogen (lower lines) and ionized hydrogen (upper lines) as a function of radius for a HII region calculation. The different lines compare the results from different numerical codes (deviations are due to the fact that different codes use different approximations to solve the radiative transfer equation and different values for the recombination coefficient and cross-section). From Iliev et al., 2000

d) Assuming photoionization equilibrium holds at each radius r, Derive an expression for the fraction of neutral hydrogen $x_{\rm HI}(r) = n_{\rm HI}/n_{\rm H}$ as a function for radius. Check that you get the right limits at some finite r when $L \to 0$ and $L \to \infty$.

e) Show that in the limit of small radii ($r \ll R_s$) there is a simple expression for the neutral fraction which gives the scaling $x_{\rm HI} \propto nL^{-1}r^2$

Comment: Figure 1 shows the ionization structure for a similar setup, calculated using full radiation transport codes, which should be qualitatively similar to your analytic result, at least for the inner regions. At the edge of the Strogmren sphere, where the matter begins to go neutral and the optical depth begins to get large, there is a very sharp transition to neutral gas.

Chilling in the Halo

A VERY SIMPLE PICTURE OF GALAXY FORMATION considers the accumulation of baryonic gas in a dark matter halo. As gas falls into a halo, it may be shocked to temperatures near the virial temperature, T_v, set by the gravitational potential

$$kT_v \sim GMm_p/R \tag{2}$$

Where M is the mass of the system and R the radius. If this shock heated gas can cool on a dynamical timescale, it may lose pressure support and condense into a galaxy[4]. The timescale for gravitational collapse is

[4] Presumably the contraction will eventually be stopped by the forming of a rotationally-supported disk or by fragmentation into stars.

$$t_{\text{dyn}} \sim \left[\frac{GM}{R^3}\right]^{-1/2} \tag{3}$$

In the largest halos, the gas will have virial temperatures $T_v > 10^7$ K and hydrogen will be completely ionized. In this case, free-free emission dominates the radiative cooling and we can make some very simple estimates of the size of the largest physical structures that may form galaxies.

a) Assume that gas in a massive halo is of constant density, spherically symmetric, and has a total mass of order the dark matter mass. If the cooling is due entirely from free-free emission from ionized hydrogen, show that the condition that the radiative cooling time is shorter than the dynamical timescale sets an upper limit on the radii of the largest structures that can collapse: $R_g \sim 80$ kpc. This simplistic estimate is actually a pretty reasonable upper limit on the size of massive galaxies in the Universe.

The real dynamics of galaxy formation are of course very complicated, and sophisticated 3-dimensional simulations are needed. Recent studies suggest that not all gas is shocked to the virial temperatures, rather much of the infall comes from narrow streams of cool ($T \sim 10^4$ K) dense gas, which are strung along filaments of the cosmic web of dark matter (see Figure 2). In addition to gravitational infall, one should also consider feedback from a variety of sources (e.g., energy injection from stars, supernovae, and AGN) which may significantly affect the dynamics. But whatever the complexities, a realistic description of radiative cooling is an essential component of any galaxy formation simulation.

Several papers have calculated and tabulated the cooling of astrophysical plasmas due to the variety of radiative processes (free-free,

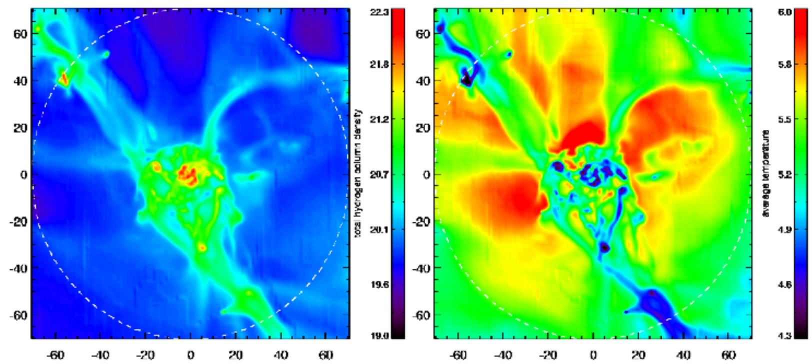

Figure 2: Simulation of a galaxy forming in a dark matter halo at redshift $z = 2.3$. The left plot shows the column density of gas, and the right plot the (density weighted) gas temperature. The dashed circle marks the virial radius of 72 kpc. One notices narrow streams of cool gas feeding the galactic disk (viewed here nearly face on). The streams are imbedded in a lower density medium of hot gas that has been shocked to the virial temperature.

bound-free, bound-bound emission). A common assumption is that the gas is in collisional ionization equilibrium (CIE) and that it is optically thin, so that any radiation produced escapes without being reabsorbed. The collisional processes that lead to cooling scale as $n_e \times n_H$, where n_e is the electron number density and n_H is the hydrogen gas density. Thus the power emitted per unit volume can be written

$$\epsilon(T) = n_e n_H \Lambda(T) \tag{4}$$

where $\Lambda(T)$ (units ergs s^{-1} cm^3) is a volumetric cooling function (or cooling coefficient). The figure in the margin shows results from one frequently referenced study. Because such cooling functions are quite important and invoked in a variety of astrophysical simulations, let's try to understand and reproduce the general features of the curve.

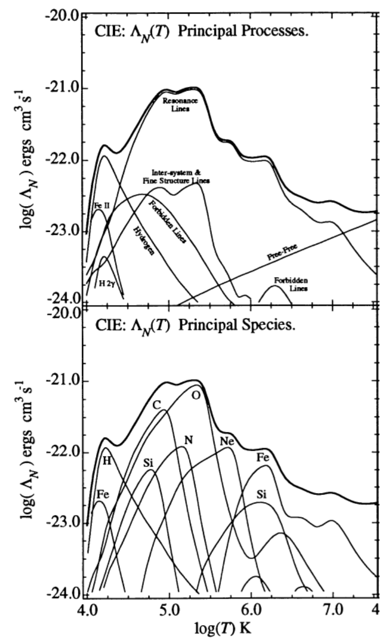

Figure 3: Cooling function for low density astrophysical plasma from Sutherland and Dopita (1993) . The top panel shows the contributions from the different radiative processes, while the bottom panel shows the contributions from different elements.

b) First, consider gas composed of pure hydrogen. Under the assumption of CIE, calculate the contributions to $\Lambda(T)$ from (1) free-free emission and (2) collisionally excited Lyman alpha line emission. Look at the temperature range, $T = 10^4 - 10^8$ K and plot (on a log-log scale like Figure 3) the cooling functions for each of the two contributions. You can use the approximate collisional ionization and excitation rates provided on the website .

c) Our optically thin assumption is reasonable for free-free and (some) bound-free emission, but can easily fail for Lyman alpha line photons[5]. However, the probability that a photon is actually reabsorbed in the L_α line is very small. Write down (in terms of the Einstein coefficients C_{21} and A_{21}) the probability that a L_α photon is absorbed into the thermal pool during a single line interaction. Show that for $n \sim 1$ cm^{-3}, $T \sim 10^4$ K, the probability of such absorption is small. The assumption of optically thin line cooling is therefore usually OK – the emitted L_α photons may scatter in the line many times,

[5] You should be able to easily show that the optical depth at line center for Lyman alpha can be as large as $\tau \sim 10^{10}$ for a gas cloud of radius 100 kpc, density 1 cm^{-3}, and assuming that some significant fraction of the gas remains cool $T \sim 10^4$ K and neutral.

but will eventually escape the system without ever really reheating the gas.

Your pure hydrogen calculation gives decent results for the lowest and highest temperatures considered, but it is clear from the Sutherland and Dopita figure that emission from metals dominates in the intermediate range $T \sim 10^5 - 10^6$ K. In particular, collisionally excited emission from resonance lines (i.e., lines where the lower level is the ground state) of metals is very important. This suggests that the dynamics of cooling systems may be sensitive to metallicity.

Rather than try to reproduce the cooling function for all metals, let's just consider one of the most important species, oxygen, emitting from the ionization state OVI. We'll continue to assume that collisional ionization and radiative recombination dominate the bound-free transition rates.

d) To determine the fractional abundance of OVI in the plasma, consider the bound-free transitions between the three ionization states, OV, OVI, and OVII (assume for now that the abundances of all other ionization stages of oxygen are zero). The relevant rates are thus the collisional ones (C_{56}, C_{67}) and the radiative ones (R_{65}, R_{76}) where, e.g., C_{56} is the collisional ionization rate from OV to OVI. Write down an analytic expression for the fraction of OVI in terms of these four rates.

e) Go to the NIST website and have it make a Grotrian diagram for OVI. Identify the wavelength of the one resonance line that you think will be most important for cooling. Take a look at the Grotrian diagrams for other ionization stages of oxygen and argue that resonant line cooling from OVII and OVIII will likely be less important, but that line cooling from lower ionization states like OV will likely make a significant contribution.

f) Calculate the cooling function from collisional excited emission of the OVI resonance line found in **e)** and add it to your plot. In doing this calculation you can assume that hydrogen is fully ionized so that $n_e = n_H$. Take the metallicity to be solar.

Comment Your cooling function should now be one step closer to the one of Sutherland and Dopita, and you can imagine adding in the contributions of other metals and ionization states to fill in the curve. Naturally, the published curves use more accurate expressions for the transition rates than we have, and include a more complete list of lines. The Sutherland and Dopita paper and other papers also consider deviations from CIE due to photoionization, which is often an important factor in galaxy formation.

Comment You can see from your cooling curve that our assumption in part **a)** that free-free cooling dominates is OK for $T > 10^7$ K, which is appropriate for the shocked gas in the most massive systems. But if we have cooler, unshocked gas in the halo, or are considering less massive galaxies with $T_v < 10^7$ K, it will be important to include other radiative processes. Clearly the cooling gas will depend strongly on the metallicity, in a way that you could presumably calculate (see Figure 4). The narrow streams of cool dense gas seen in Figure 2 have $T \sim 10^4$ K and so should be effective Lyman alpha line emitters. In fact, looking for line emission from the circumgalactic medium is one good way to test the predictions of these simulations.

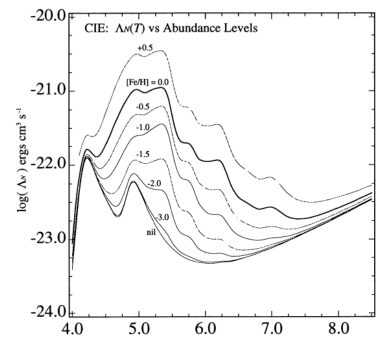

Figure 4: Cooling function for low density astrophysical plasma for different metallicities; from Sutherland and Dopita (1993) .

Comment Somewhat similar arguments about gravitational infall and cooling appear in modeling the formation of molecular clouds and stars. Our cooling curve so far is only reasonable for temperatures around 10^4 K or above. At lower temperatures, cooling from dust and molecular line emission may start to play a significant role. Note also that if the system becomes dense enough, the assumption of optically thin cooling may no longer hold, and one would have to solve the radiative transfer equation to determine the rate at which radiation actually escapes the system. In 3-D simulations, this is often done using the flux-limited diffusion approximation.